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Supergravity and worldvolume physics in the dual description of $\mathcal{N} = 1^*$ theory

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Abstract: We look at the string theory dual of the $\mathcal{N} = 1^*$ theory, involving 5-branes, which was recently proposed by Polchinski and Strassler [1]. We argue that SUGRA alone is not enough in order to obtain the correct screening and confinement behaviour of the various massive field theory vacua, but that appropriate worldvolume phenomena of the 5-branes must be included. We therefore work within the SUGRA approximation, also taking into account the brane dynamics, and classify all the SUGRA configurations. In this level of analysis, we find multiple valid configurations for every given vacuum. We discuss some possible resolutions of this perplexing result. We also consider the spectrum of asymptotic states, and discuss the global symmetries of the SUGRA solution of the $\mathcal{N} = 1^*$ theory and of the $\mathcal{N} = 0^*$ theory obtained from it by explicit supersymmetry breaking.

Keywords: AdS-CFT Correspondance, Brane Dynamics in Gauge Theories, Confinement.

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Contents

1. Introduction

The AdS/CFT correspondence [2, 3, 4] was originally formulated for theories with 16 supersymmetries. A string theory dual of a non-conformal confining theory with 4 supersymmetries was recently suggested by Polchinski and Strassler [1]. This theory has come to be known as the $\mathcal{N} = 1^*$ theory. It is derived from the $\mathcal{N} = 4$ theory by adding mass terms to the three $\mathcal{N} = 1$ chiral multiplets.

The supergravity dual suggested in [1] is obtained by turning on the dual supergravity modes of the mass terms added. This results in the creation of a 5-brane wrapped on an S^2. A five dimensional gauged SUGRA dual of the same theory was presented in [5]. Although the supergravity equations were only solved to first order in the perturbation, certain qualitative features emerge. The most striking feature that can, according to [1], be obtained from the supergravity dual is the existence of confinement in some of the vacua of this theory. Other related works that followed are [6]–[13].

The purpose of this paper is to point out some subtleties in the duality between the $\mathcal{N}=1^*$ theory and the supergravity description suggested in [1]. In section 2 we review the main features of the $\mathcal{N}=1^*$ theory from the field-theoretic point of view as well as from the one of the SUGRA dual solution.

In section 3 we address the inadequacy of SUGRA to describe the screening of charges in the dual field theory. It was argued in [1] that using the SUGRA duals of the various vacua of the field theory, one can manifest screening, or the existence of confinement with the appropriate string tension. The configuration of p D5-branes corresponds to the unbroken $\mathrm{SU}(p)$ vacuum of $\mathrm{SU}(N)$ (with $p|N$). We show that except for the special case $p=1$ (a Higgs vacuum) discussed in [1], one needs to incorporate the worldvolume theory on the 5-branes in order to get the right set of confined and screened charges. A purely SUGRA analysis leads to the wrong conclusion of total screening in *all* of those vacua. Analogously, a purely SUGRA analysis of q NS5-branes configuration (with $q|N$) gives an erroneous result of monopole screening instead of the correct $\mathrm{SU}(N/q)$ vacuum.

Polchinski and Strassler have argued that within their approximation there is only one valid SUGRA description for each field theory vacuum. In section 4 we find and describe the multiplicity of SUGRA descriptions of each field theory vacuum. We argue that, in this level of analysis, there is a multitude of valid descriptions for each such vacuum. We then discuss some possible resolutions of this perplexing result. Some details are given in the appendix.

In section 5 we consider the asymptotic states in the theory. We do not calculate the exact glueball spectrum, but work with the simplified analogue of a minimally coupled scalar. This is intended to give a rough idea of the glueballs mass scale. Section 6 deals with the global symmetries of the theory and the possibilities for complete breaking of supersymmetry. Finally, section 7 is a summary and discussion of our results.

2. A review of $\mathcal{N}=1^*$ theory

The starting point is the $\mathcal{N}=4$ SYM theory, whose field content is — in the $\mathcal{N}=1$ language — three chiral superfields Φ_i, $(i=1,2,3)$, and a vector superfield, all in the adjoint representation of the gauge group. The superpotential is

$$W=\frac{\sqrt{2}}{3g_{YM}^2}\,\mathrm{tr}\left(\epsilon^{ijk}\Phi_i\left[\Phi_j,\Phi_k\right]\right). \tag{2.1}$$

The theory possesses $\mathrm{SO}(6)\simeq\mathrm{SU}(4)$ R-symmetry of which $\mathrm{SU}(3)$ (rotating the three chiral superfields) and $\mathrm{U}(1)$ (the $\mathcal{N}=1$ R-symmetry) are manifest in the superpotential. The deformations which were considered are the turning on of mass terms — either a mass m_4 for the gaugino, in which case supersymmetry is completely broken, or masses m_1, m_2, m_3 for the chiral superfields, keeping $\mathcal{N}=1$ supersymmetry. The latter deformation is performed by adding to the superpotential

$$\Delta W = \frac{1}{g_{YM}^2}\left(m_1 \operatorname{tr}\Phi_1^2 + m_2 \operatorname{tr}\Phi_2^2 + m_3 \operatorname{tr}\Phi_3^2\right), \tag{2.2}$$

thus breaking the R-symmetry down to U(1) (which is further broken by anomaly). An interesting and simpler case is the SO(3) invariant theory where all masses are chosen to be equal. Ideally, one would like to take m_i to infinity, thus decoupling the chiral superfields and ending up with a pure $\mathcal{N}=1$ SYM. However, this cannot be done within the AdS/CFT correspondence since the dynamically generated energy scale is

$$\Lambda^3 = m_1 m_2 m_3 \exp\left(\frac{2\pi i\tau}{N}\right) \sim m_1 m_2 m_3 \exp\left(\frac{-8\pi^2}{\lambda}\right), \tag{2.3}$$

(where $\tau = \theta/2\pi + 4\pi i/g_{YM}^2$, and $\lambda = g_{YM}^2 N = 4\pi g N$ is the 't Hooft coupling constant), and decoupling can only be achieved with $\lambda \to 0$ which invalidates the SUGRA approximation. One has, therefore, to deal with the $\mathcal{N}=1^*$ theory in which the fermions and scalars are not decoupled. Considering the string theory dual of this perturbation, the fermion mass matrix transforms as the $\overline{\mathbf{10}}$ of SU(4) and was shown [1] to correspond to the lowest S^5 spherical harmonic, non-normalizable mode of the 3-form $G_3 = F_3 - \tau H_3$ on $AdS_5 \times S^5$. The asymptotic behavior of this form should correspond to a tensor that transforms appropriately under SO(6). Such a tensor can be formed by defining complex coordinates

$$z_i \equiv \frac{x_{i+3} + i x_{i+6}}{\sqrt{2}} \equiv \frac{w_i + i y_i}{\sqrt{2}} \tag{2.4}$$

and regarding the form

$$T_3 = m_1 dz_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3 + m_2 d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_3 + m_3 d\bar{z}_1 \wedge d\bar{z}_2 \wedge dz_3 + m_4 dz_1 \wedge dz_2 \wedge dz_3 \tag{2.5}$$

and descendents thereof, where the x_i's are the six coordinates transverse to the D3-branes.

2.1 Field theory vacua

The $\mathcal{N}=1^*$ theory has been investigated by Vafa and Witten who have found the classical vacua [14], and further by Donagi and Witten who have considered the quantum vacua [15]. A supersymmetric vacuum must have zero F- and D- terms. Assuming the three masses m_1, m_2, m_3 are nonzero, the chiral superfields may be rescaled such that all masses are equal to m, and one finds

$$\begin{aligned} F_i &= \frac{m}{\sqrt{2}}\phi_i + \epsilon_{ijk}[\phi_j, \phi_k] \\ D &= \sum_{i=1}^{3}[\phi_i^\dagger, \phi_i], \end{aligned} \tag{2.6}$$

where the ϕ's are the lowest (scalar) components of the chiral superfields. The F-term equations dictate that a vacuum is determined by an N dimensional representation of SU(2), i.e., a homomorphism from the algebra of SU(2) to the one of the gauge group SU(N). These vacua are to be modded out by the gauge transformations and imposed with the D-term condition. This amounts to modding by the complexified gauge group SL($N, \mathbb{C}$), or, alternatively, considering the conjugacy classes of the homomorphisms. As SU(2) has, up to conjugacy, a single irreducible representation of every dimension, we end up with a discrete set of vacua (corresponding to the choice of the *reducible* representations we make). The apparent SO(3) global symmetry is equivalent to a gauge transformation of an SO(3) $\subset$ SU(N). This can also be shown explicitly — consider an infinitesimal SO(3) transformation, e.g. a rotation in the $1-2$ plane

$$\delta \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} \varepsilon\phi_2 \\ -\varepsilon\phi_1 \\ 0 \end{pmatrix} . \tag{2.7}$$

This transformation can also be performed (around a classical vacuum) by commuting with ϕ_3, which by the D-term condition may be taken as hermitean, thus being a generator of SU(N). This can also be done for ϕ_1 and ϕ_2, showing that the global SO(3) mixes with the gauge group. Therefore, in each of the (non confining) vacua there is a new SO(3), which is a combination of the original SO(3) and the gauge group. This new global symmetry will presumably be the isometry of the SUGRA description of that vacuum.

To conclude, in the classical analysis, each of the discrete set of vacua of the $\mathcal{N}=1^*$ theory is described by assigning, as *vevs* of the scalars, an N-dimensional, generically reducible, representation of SU(2). Using the above mentioned uniqueness of the *irreducible* representations, each vacuum corresponds to a partition of N into positive integers. The resulting unbroken gauge group of the theory depends upon this choice, varying from the full SU(N) confining phase (corresponding to N copies of the trivial representation, i.e. $\phi_i = 0$, $i = 1,2,3$) to the completely Higgsed phase (The irreducible N-dimensional representation). The massive vacua, those with a mass gap, correspond to divisors of N, $N = p \cdot q$, where the unbroken gauge group SU(p) rotates the p copies of the q-dimensional representation. A partition $N = \sum_{d=1}^{N} d\, k_d$ with more than one nonzero k_d corresponds to a Coulomb vacuum where the gauge group includes a U(1) factor and there is no mass gap.

Classification of the quantum vacua is more complex. Considering only the massive vacua, the gaugino condensate $\langle\lambda_4\lambda_4\rangle$ splits each SU(p) vacuum into p different quantum vacua. Therefore, the number of quantum massive vacua of the SU(N) $\mathcal{N}=1^*$ theory is given by the sum of the divisors of N, which we shall denote by $\sigma_1(N) \equiv \sum_{p|N} p$. These vacua are in a one-to-one correspondence with 't Hooft classification [16]: consider the lattice $L \equiv \mathbb{Z}_N \times \mathbb{Z}_N$ of external electric and magnetic charges (one factor corresponds to the center of the gauge group while the other to the fundamental group $\pi_1[\mathrm{SU}(N)/\mathbb{Z}_N]$). Each vacuum is characterized by a subgroup P of the condensed charges. The massive vacua correspond to subgroups of order N, and in these vacua the screened charges are exactly the condensed ones. In the confining vacuum, the magnetic monopoles are condensed and the subgroup is generated by $(0,1)$, while in the Higgs vacuum the condensed charges are the electric ones, and the subgroup is generated by $(1,0)$. In general, an $\mathrm{SU}(p)$ vacuum corresponds to a subgroup generated by $(p,0)$ and (s,q), with $s=0,\ldots,p-1$ distinguishing between confinement and various oblique confinements. This classification exhausts the order N subgroups, and also clarifies the action of the $\mathrm{SL}(2,\mathbb{Z})$ duality which is inherited from the Montonen-Olive symmetry in $\mathcal{N}=4$ SYM. The elements of the duality group $\mathrm{SL}(2,\mathbb{Z}_N)$ act in the natural way on elements of L, permuting between the various massive vacua.

The complete superpotential of $\mathcal{N}=1^*$ theory, including quantum corrections, has been computed [17], and it was shown that the aforementioned enumeration of vacua still holds.

We can give a simple physical picture of confinement. With $N=p\cdot q$, the adj representation of $\mathrm{U}(N)$ decomposes to (adj, adj) in the obvious $\mathrm{U}(p)\times\mathrm{U}(q)\subset\mathrm{U}(N)$. For $\mathrm{SU}(N)$ the adj decomposes into $(\mathrm{adj},\mathrm{adj})\oplus(1,\mathrm{adj})\oplus(\mathrm{adj},1)$ of $\mathrm{SU}(p)\times\mathrm{SU}(q)\subset\mathrm{SU}(N)$. When we further break the $\mathrm{SU}(q)$ factor, it decomposes to q^2 copies of the $\mathrm{SU}(p)$ adj and to singlets. For the massive vacua the gluons of the unperturbed theory do not produce fundamental representations. It follows that external fundamental charges ("massive quarks") cannot be screened by pair creation from the vacuum. In conclusion, for $1<p$, $p|N$, we indeed expect confinement of basic electric charges in the $\mathrm{SU}(p)$ vacuum.

2.2 String dual description

As mentioned before, the mass deformation corresponds to turning on the SUGRA form G_3. According to Myers' dielectric brane mechanism [18], this results in an induced 5-brane dipole, or more specifically, considering the equal masses case, a 5-brane which wraps an S^2 equator of the unperturbed S^5 at a constant AdS radius and also spans the four dimensional Minkowski space. This 5-brane has a D3 charge distribution on it due to the original D3-branes. Different massive vacua correspond to different kinds of wrapped 5-branes. The location (in the AdS radius) of these branes is determined by the minimum of the brane action, which was calculated by Polchinski and Strassler [1] to first order in a small parameter $g|M|^2/N$ (where $M=c\tau+d$ for a (c,d) 5-brane). The complete metric and dilaton background of these vacua were also derived, as well as the solution of G_3.

The $\mathrm{SU}(p)$ vacuum, with $N=p\cdot q$, was described by two alternative dual configurations - either p coincident D5-branes or q coincident NS5-branes. The first description seems plausible, because of the low energy theory of open strings ending on the D5-branes, but the second description is more mysterious. The complete metric for the p D5 solution is

$$ds^2 = Z^{-1/2}dx_{0123}^2 + Z^{1/2}(dy^2 + y^2 d\Omega_y^2 + dw^2) + Z^{1/2}\Omega^{1/2}w^2 d\Omega_w^2 \qquad (2.8)$$

(corresponding to a brane located at $|\vec{w}| = r_0$ and $|\vec{y}| = 0$ where $\vec{w}, \vec{y}$ are both three dimensional) with

$$Z \equiv \frac{R^4}{\rho_+^2 \rho_-^2}\,, \qquad \Omega = \left[\frac{\rho_-^2}{\rho_-^2 + \rho_c^2}\right]^2$$

where, for 't Hooft coupling λ of the unperturbed theory,

$$\frac{R^4}{\alpha'^2} = \lambda\,, \qquad \rho_\pm = \left(y^2 + [w \pm r_0]^2\right)^{1/2}, \qquad \rho_c = \frac{\alpha'^2 R^2 m}{2}\,, \qquad r_0 = \pi\alpha' m q\,.$$

The nontrivial dilaton is

$$e^{2\phi} = g^2 \frac{\rho_-^2}{\rho_-^2 + \rho_c^2}\,.$$

The conditions for this solution to be valid are: a small string coupling constant, small curvature (i.e. large radius of the transverse sphere) and a small perturbation around the solution with $m = 0$ (the small parameter mentioned above). For the two descriptions of SU(p) with zero θ-angle, the conditions are given in table 1.

	p D5's	q NS5's
small perturbation	$q \gg gp$	$q \ll gp$
small coupling	$g \ll 1$	$g \ll 1$
large transverse sphere	$gp \gg 1$	$q \gg 1$

Table 1: Validity conditions of the solutions for the SU(p) vacuum.

The ranges of validity of these two descriptions have no overlap, but for each large enough value of λ there are valid solutions considering both descriptions.

3. Screening and confinement via stringy effects

Polchinski and Strassler look at the behavior of Wilson loops for the fully Higgsed vacuum, using the single D5-brane vacuum, although the SUGRA solution does not obey the aforementioned conditions. They find complete screening, as they expect. We shall now claim that the SUGRA description is inadequate even for the "electric" case (fundamental strings ending on D5-branes), and that stringy effects of the brane world-volume theory must be taken into account.

3.1 Wilson-'t Hooft loops as string probes approaching 5-branes

Let us first briefly recall the case of flat D5-branes. The metric is given by

$$ds^2 = \frac{\rho}{R'}dx_{012345}^2 + \frac{R'}{\rho}d\rho^2 + R'\rho\, d\Omega_3^2\,. \qquad (3.1)$$

An open string probe in this background, with both ends fixed on the Minkowski boundary of (3.1), gives the dual description of the Wilson loop of a quark anti-quark pair [19, 20]. A simple scaling argument [21] shows that a smooth configuration obeying the equations of motion must have the quarks at a fixed distance apart (this distance is a numerical multiple of R'). For smaller distances, the string will be pulled to the boundary ($\rho \to \infty$) and there is no stable solution. For larger distances only the non smooth configuration of two "bare" quarks, represented by two straight "vertical" strings from the boundary to the origin, is allowed. This configuration corresponds to a *screening* behavior. This correspondence can be understood in the following manner. Consider a "horizontal" string segment connecting the two quark lines at some given value of $\rho = \rho_0$. This segment has a Nambu-Goto action which is linear with the separation distance and with a "string tension" given in the SUGRA approximation as

$$\sigma(\rho_0) = {\alpha'}^{-1} \sqrt{G_{tt} G_{xx}} = \frac{\rho_0}{\alpha' R'} \, . \tag{3.2}$$

If the SUGRA picture is valid all the way down to $\rho \to 0$ it is clear that one finds a zero string tension, namely, screening. However, this SUGRA analysis cannot give the complete answer. For $\rho \sim \alpha'/R'$, the curvature becomes large in string units, and the SUGRA solution and description of the Wilson loop ceases to be valid. The smallest reliable tension SUGRA can give us is $\sigma_0 = \sigma(\alpha'/R') \sim 1/{R'}^2$. The question what is the tension when the string probe reaches smaller values of ρ, and whether it vanishes for $\rho \to 0$, cannot be answered by SUGRA alone.

This conclusion holds for any number of flat D5-branes, as their number only enters through R'.

We now turn to the actual metric (2.8) of the 5-branes wrapped on S^2. Following [1], we analyze the Nambu-Goto action, ignoring the contribution of the NS potential. Very far from the sphere, the D3 charge dominates and the metric is asymptotically $AdS_5 \times S^5$. This, of course, allows for small distance Wilson loops, in contrary to the flat D5 case. Near the sphere, however, the metric resembles that of a stack of flat D5-branes. Indeed, as an expansion in small ρ_- shows, ρ_- plays the role of ρ in (3.1), the surface of the sphere seems flat, and the two coordinates of $d\Omega_w$, properly scaled, join the four dimensional Minkowski space to form a six dimensional one, while the extra angle between y and $w - r_0$ joins the two coordinates of $d\Omega_y$ to form $d\Omega_3$. We also have $R' = R^2/r_0$. We therefore expect that in the naive SUGRA analysis there is a maximal value for the distance L between the ends of a smooth Wilson loop. As the quarks are moved farther apart, the string touches the stack of branes and the favored configuration becomes necessarily that of two bare quarks, as in the flat D5 case. The exact shape of the Wilson loop depends on the boundary condition for its position in the w, y plane, but at least for the case of the loops with $y = 0$, $r_0 \leq w \leq \infty$, this is proven by the theorems in [22]. The analysis is also performed for a similar system in [23]. Therefore, the proposed SUGRA picture shows *complete screening*, even for the SU(p) vacuum, where we expect *confinement* of the basic electric charges. Hence, the correct description necessitates stringy effects and we definitely cannot rely on SUGRA to discern between confinement and screening!

However, the minimum tension for a bit of string, artificially placed "horizontally" in a region where SUGRA *is* reliable, gives an upper bound for the actual string tension. This provides us with a consistency check of the proposed duality. The minimum reliable SUGRA tension takes the value $\sigma_0 \sim r_0^2/R^4 \propto m^2 \cdot q/gp$. We remember that the necessary condition for the metric to be valid as a first order approximation, i.e. for the perturbation to be small, is $q \gg gp$. We therefore have $\sigma_0 \gg m^2$. On the other hand, all the three consistency conditions listed above imply $\lambda \gg 1$ and therefore, by (2.3), $\Lambda_{YM} \sim m \exp(-8\pi^2/3\lambda) \approx m$. We expect the string tension, if confinement occurs as in the pure $\mathcal{N} = 1$ case, to be of order $\Lambda_{YM}^2 \approx m^2$. We indeed find that the upper bound σ_0 is higher than that.

The dual description of the SU(p) vacuum, using NS5-branes, can be seen, in an analogous way to the D5 description, to give complete monopole screening from the SUGRA analysis. Again, only stringy effects can lead to the correct SU(p) vacuum confinement of basic magnetic monopoles.

3.2 Strings ending on branes and "electric" screening

The field-theoretic picture of the SU(p) vacuum of $\mathcal{N} = 1^*$ demands that there is confinement of basic electric charges. We have shown that SUGRA alone cannot account for this phenomenon. A full understanding of string theory in regions of spacetime which include branes, where the curvature is high and there are other strong SUGRA fields, should provide the correct answer. Until such an understanding is available, we should assume some effects associated with the 5-branes worldvolume physics.

In the picture of p wrapped D5-branes of the SU(p) vacuum, a $(1, 0)$ string (a fundamental string) ending on these branes is a quark of the SU(p) worldvolume theory on the p D5-branes. A non abelian flux tube inside the branes should confine a quark anti-quark pair into a singlet (a meson). Such a singlet is trivial from the point of view of the SU(N) theory. However, p fundamental strings form a singlet of the SU(p) worldvolume theory (an "electric" baryon), and this corresponds to a non trivial screening of $(p, 0)$ charges in the SU(N) theory, and the breaking of the SU(N) group down to SU(p).

This picture can be generalized using the SL$(2, \mathbb{Z})$ symmetry of the IIB theory. On a wrapped stack of $(c, d) = p' \cdot (c', d')$ 5-branes, where $p'|N$ and c', d' are mutually prime, a (d', c') string may naively end (in our conventions), but only a bunch of p' such strings can indeed do so. For q NS5-branes (with $N = p{\cdot}q$), this means screening of q D1 strings, or of q monopoles, as required in the non oblique SU(p) vacuum.

However, there is a subtlety here. Suppose there is a wrapped (c, d) brane with $f = \gcd(c, d)$, $p' = \gcd(f, N)$. Usually such a configuration is viewed as f coincident branes, but we claim that in our scenario it should be viewed as p' coincident branes. Indeed, the dielectric effect gives the branes a common radius r_0 at equilibrium. r_0 is proportional to the ratio of D3-brane to 5-brane charges. For any radius different from r_0, there is a positive energy density of the configuration and supersymmetry is completely broken. However, the D3 charge, N, embedded in these 5-branes cannot be equally divided between f-branes. There is an energy barrier between the one-shell configuration and any configuration of f displaced branes (a stable such configuration, where the branes form several separated shells, necessarily corresponds to a Coulomb vacuum). The energy barrier per volume is proportional to N, so cannot be overlooked in the $N \to \infty$ limit, and is infinite for the infinite volume limit of the field theory. We conclude that a (c', d') 5-brane is to be considered a single brane, if its D3 charge is mutually prime with $\gcd(c', d')$, even if the latter is bigger than one. For "electric" screening this means that the (c, d) 5-brane allows screening of p' copies of $(d'', c'') \equiv (d/f, c/f)$ strings (or charges). Note that this result is also true modulu N.

Such configurations, in which the D3 charge cannot be evenly split between the (naively) multiple 5-branes, may seem superfluous. They are present in the SUGRA analysis, but, of course, in the full string-theoretic treatment they might either disappear or display different dynamics. We return to this point at the end of the next section.

3.3 Baryon vertices and "magnetic" screening

Polchinski and Strassler describe D3-branes filling the S^2 sphere on which the 5-branes are wrapped, which behave as baryon vertices. These baryon vertices arise through the Hanany-Witten effect [24], when a baryon vertex of the unperturbed $\mathcal{N} = 4$ theory, which is a 5-brane wrapping the S^5, contracts and moves through the 5-brane S^2 shell. For the $\mathrm{SU}(p)$ vacuum described by p wrapped D5-branes, each D5-brane has a dissolved D3 charge of q, and its analysis shows that the junction of a D3 ball with a D5 wrapped on S^2 must support strings with total $D1$ charge of q. Polchinski and Strassler do not constrain the $F1$ charge of the strings, but simply take it as zero. This mechanism explains the "magnetic" screening of q monopoles in the non oblique $\mathrm{SU}(p)$ vacuum. We shall presently show that the complete picture is more involved.

We work in the convention where an element of $\mathrm{SL}(2, \mathbb{Z})$, represented by a matrix $M = \left(\begin{smallmatrix} \alpha & \beta \\ \gamma & \delta \end{smallmatrix}\right)$, operates on the charge vector $(e, m)^t$ or the string vector $(\mathrm{F1}, \mathrm{D1})^t$ by multiplication, while on the 5-brane vector $(\mathrm{NS5}, \mathrm{D5})^t$ it operates by a multiplication by $(M^{-1})^t = \left(\begin{smallmatrix} \delta & -\beta \\ -\gamma & \alpha \end{smallmatrix}\right)$. We take the generator T of $\mathrm{SL}(2, \mathbb{Z})$ to be given by $T = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$ and denote $T' = S^{-1}TS = (T^{-1})^t = \left(\begin{smallmatrix} 1 & 0 \\ -1 & 1 \end{smallmatrix}\right)$.

Let us look at the p D5-branes description of the non oblique $\mathrm{SU}(p)$ vacuum. By performing a $(T')^b$ action, every D5-brane becomes a $(b, 1)$ 5-brane, and the "electric" screening is now of $p \cdot (1, b)$ charges. This amounts to changing to some oblique vacuum. Let us now apply the same transformation to the q NS5-branes description of the non oblique $\mathrm{SU}(p)$ vacuum. This has no effect on NS5-branes, but the vacuum must transform in the same manner as before!

The answer, of course, is that the "magnetic" screening is changed. The $p\cdot(1,0)$ strings on the baryon vertex of the NS5 *are* affected by the action of $(T')^b$, and transform into $p\cdot(1,b)$ strings, giving the same vacuum as before. This example shows that the field-theoretic vacuum is *not* completely described by the 5-brane wrapping the S^2. As $(p,0)$ charges are screened "electrically" by the p D5 wrapped branes, the charges screened "magnetically" can be any of (b,q), with the relevant values of b being $0 \le b < p$. These p possibilities correspond to the splitting of the classical SU(p) vacuum into p quantum vacua, parameterized by the expectation values of the gaugino bilinear $\langle\lambda_4\lambda_4\rangle$. It cannot be said that the p D5-brane configuration is canonically the non oblique SU(p) vacuum — all the SU(p) oblique vacua are on equal footing, and the condensation choosing between them is another worldvolume phenomenon that cannot be inferred from the SUGRA solution alone. Note, however, that the action of $(T')^b$ also changes the NS potential $B_{\mu\nu}$ (G_3 is invariant as τ is also changed).

A $p'\cdot(h,0)$ 5-brane configuration representing p'-branes (i.e. having $\gcd(h,q')=1$ with $p'\cdot q' = N$), will give "magnetic" screening of $q'\cdot(1,0)$ charges. This can be seen from the analysis of Polchinski and Strassler, remembering that the baryon vertex, arising from the Hanany-Witten effect, will consist of h D3-branes filling the sphere, and those D3-branes can separate in the Minkowski space, each carrying q' fundamental strings.

Generally, a $p'\cdot(c',d')$ configuration can be obtained from a $p'\cdot(h,0)$ one using multiplication by

$$\begin{pmatrix} c'' & r'' \\ d'' & s'' \end{pmatrix} \equiv \begin{pmatrix} c'/h & r'/h \\ d'/h & s'/h \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) \tag{3.3}$$

and then the "magnetic" screening will be of $q'\cdot(s'',r'')$ charges.

3.4 Compatibility with the $\mathbb{Z}_N\times\mathbb{Z}_N$ lattice structure

We have seen that a $(c,d) = p'\cdot(c',d')$ 5-brane configuration, with q' and $h = \gcd(c',d')$ mutually prime, has two generators of the screened charges lattice — the "electric" one $p'\cdot(d'',c'') \equiv p'\cdot(d'/h,c'/h)$, and the "magnetic" one is some $q'\cdot(s'',r'')$ where $p'\cdot q' = N$ and (3.3) holds.

First we note that this solution gives screening compatible with the $\mathbb{Z}_N\times\mathbb{Z}_N$ structure of the field theory charges lattice. Indeed, from (3.3), the two vectors (c'',d'') , (r'',s'') span the whole $\mathbb{Z}\times\mathbb{Z}$ lattice, so the two aforementioned generators certainly span $(N,0)$ and $(0,N)$. Therefore, we may concentrate on the $\mathbb{Z}_N\times\mathbb{Z}_N$ lattice spanned by the two generators modulu N. We summarize the configuration in the matrix

$$\begin{pmatrix} c'' & d'' \\ r'' & s'' \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}_N)\,, \tag{3.4}$$

where we also attach to this matrix the number $p'\in\mathbb{Z}_N$.

Next, if there is some combination of the generators giving a trivial charge,

$$(ap', bq') \begin{pmatrix} c'' & d'' \\ r'' & s'' \end{pmatrix} \equiv (0,0) \pmod N, \tag{3.5}$$

then by the invertibility of the matrix, $a \equiv 0 \pmod{q'}$ and $b \equiv 0 \pmod{p'}$. The two generators therefore generate a lattice of size $p' \cdot q' = N$, corresponding indeed to a massive vacuum.

4. A classification of all brane configurations leading to a given vacuum

Now that we have a full description of the vacua generated by wrapped 5-branes, we can try and classify all the configurations giving rise to a given field theory vacuum. In this section we will restrict ourselves to square-free N, that is N which is a product of distinct primes. For such an N, if $N = p' \cdot q'$, then p' and q' are necessarily mutually prime.

Let us begin with an example involving the $\mathrm{SU}(p)$ non oblique vacuum. We know from [1] that it can be either represented by p D5-branes or by q NS5-branes. In the first description, the generator $(p, 0)$ of the screened charges subgroup arises "electrically" and the generator $(0, q)$ arises "magnetically", while the roles are reversed in the second description. However, when p and q are mutually prime, the subgroup $P = \langle (p,0)\,,\,(0,q)\rangle$ is cyclic, and is generated by a single element: $P = \langle (p,q)\rangle$ (the choice of generator is, of course, non unique). This is just the Chinese remainder theorem, or the simple assertion that in these circumstances, $\mathbb{Z}_p \times \mathbb{Z}_q = \mathbb{Z}_{pq}$. From the previous discussions it follows that the same vacuum can also be described by a single (q, p) 5-brane, where the full screened charges lattice is accounted for "electrically", and the "magnetic" screening accounts only for the modularity by N.

In fact, when N is square-free, the only abelian subgroup of order N is $\mathbb{Z}_N$, and all the massive vacua subgroups of the $\mathbb{Z}_N \times \mathbb{Z}_N$ lattice are isomorphic. When p and q are not mutually prime, the subgroup $P = \langle (p,0)\,,\,(0,q)\rangle$ is not cyclic, and therefore this does not hold for N which is not square-free.

We know that the action of T rotates between the different oblique vacua corresponding to the same electric screening. From the non oblique $\mathrm{SU}(N)$ confining vacuum we can therefore get to all the oblique analogues, where the "magnetic" screening can be $\mathrm{SU}(q)$ for all $q|N$. Analogously, given $p|N$ the action of T' can take us from the confining vacuum to some vacuum with electric $\mathrm{SU}(p)$, and then by T to any oblique or non oblique analogue of it. We see that for square-free N, the vacua are isomorphic by the action of the $\mathrm{SL}(2, \mathbb{Z}_N)$. Therefore it is enough to study the representations of a single vacuum, e.g. the confining vacuum.

When we have a (c, d)-brane, with $f = \gcd(c, d)$, we should first of all determine $p' = \gcd(f, N)$ and then $c'' = c/f$, $d'' = d/f$ (as a matter of fact, in order to obtain

c'', d'' it is sufficient, modulu N, to divide c, d only by the factor of f consisting of all the primes which divide N, to the appropriate power). These c'', d'' are obviously given modulu N. Then, r'', s'' obeying (3.4) should be found, but there are only p' such pairs giving distinct physical configurations. We can now ask the reverse question — classify all the pairs $(c'', d'') \in \mathbb{Z}_N \times \mathbb{Z}_N$, mutually prime modulu N, appropriate pairs (r'', s'') and with an attached number $p'|N$, giving a prescribed vacuum.

We find that each vacuum is obtained exactly once from every (c'', d'') pair, with an appropriate p' and (r'', s''). The number of such representations for each vacuum is $\phi(N)\sigma_1(N)$, where $\phi(N)$ is Euler's totient function, counting the number of $1 \leq i \leq N$ relatively prime to N, and $\sigma_1(N)$ is the sum of the divisors of N. The details and examples are left for the appendix.

Not all the representations we have found are small perturbations of the $\mathcal{N} = 4$ SUGRA dual solution. Indeed, for a shell of wrapped (c, d) 5-branes, with $M = c\tau + d$. The small curvature condition is then $g|M| \gg 1$, and the small perturbation condition [1] is $N \gg g|M|^2$. Using the averages inequality, for the case without θ angle, we translate the small perturbation condition to

$$N \gg g|M|^2 = 2g \cdot \frac{1}{2}\left(\left(\frac{c}{g}\right)^2 + d^2\right) \geq 2g \cdot \left(\frac{cd}{g}\right) = 2cd\,, \tag{4.1}$$

which is certainly satisfied only for a very limited part inside the periodicity range considered above. For example, the (q, p) 5-brane solution of the SU(p) vacuum is already (marginally) outside the range of validity. However, even if the perturbations are not small, the first order approximations to the SUGRA equations do not hold, and the explicit solution of [1] is not valid, wrapped 5-branes still seem to provide the correct qualitative behavior as described above.

Be that as it may, our analysis shows, for example, that for odd N, the $(0, 2)$ 5-brane configuration yields the same Higgs vacuum as a $(0, 1)$ configuration (a single D5). Certainly, when the latter description is valid, so is the former. Therefore, there inevitably are multiple valid descriptions of the same vacuum. More generally, we may multiply a valid (c, d) 5-brane configuration by a small number relatively prime to N, to get another valid description (in the appendix we also show that in a certain sense this is the generic way to get multiple valid descriptions).

The parallel in field theory of the situation described in the last paragraph is the non uniqueness of the generators of the condensed charges subgroups. In the field theory classification of vacua, a generator has no meaning, only the group does. In the the dual theory, however, different 5-brane configuration may correspond to different generators, and moreover, each generator has an infinity of 5-brane configurations giving it. This is the source of the multiple stringy descriptions of each field theory vacuum.

JHEP03(2001)013

All the above is contrary to the view of [1] that there is only one valid description for every choice of vacuum and value of g. Clearly this result is puzzling. If we believe in the AdS/CFT duality and its generalizations, we expect to find only one valid SUGRA background dual to a certain field theory (although, of course, when the parameters of the field theory change, the valid description can also change, as was shown in [1]). It is a challenging question to check whether this multi-description is indeed present. There are several possible resolutions to this puzzle:

1. The different solutions may turn out to be in fact equivalent (therefore describing exactly the same field theory). This, however, is unlikely, as it seems that the details of the physics probed by the various strings (i.e. the exact potential between external charges), as well as the physics detected by other probes (i.e. other branes and bulk fields) would be different in the different solutions.
2. The multiplicity present in the level of the brane configuration analysis might be removed in the still lacking full string theory treatment, i.e. stringy effects beyond the SUGRA approximation with brane dynamics might render some of the solutions invalid. The natural candidate solutions to be removed are those in which the D3 charge is not divisible by the "naive" number of 5-branes. This seems to us to be the most probable option.
3. Some of the superfluous solutions might correspond, through some stringy mechanism yet unknown to us (perhaps on the worldvolume of the branes), to massless vacua. As there are many such vacua, and as they are less well understood, this might help to solve the problem.
4. Even within the SUGRA approximation some of the solutions may be non-BPS and unstable. They would therefore represent valid non-vacuum field theory states. These states would have non-zero energy density and therefore would be "false vacua".
5. Following ideas [25, 26] similar to those associated with "giant gravitons" [27], it might be possible that there are instantonic tunnelings between different 5-brane configurations giving the same field theory vacuum. In this case the SUGRA solution should be taken as a superposition of these configurations. However, it is not clear what is the field-theoretic analogue of such a superposition. Moreover, there does not seem to exist in the field theory a "theta angle" corresponding to superpositions with relative phases.

5. Mass gap and asymptotic states

It was pointed out in [1] that $\mathcal{N} = 1^*$ theory has a rich spectrum of asymptotic states. Some of the expected states, like monopoles and W-bosons, cannot be seen by the

SUGRA solution presented in [1]. These require a full analysis of the worldvolume theory on the 5-branes referred to in section 3. The spectrum of states corresponding to bulk supergravity modes can be calculated in much the same way as in [28, 29, 30]. One major difference is that SO(6) non-singlet modes were not considered in those papers, since they have no corresponding states in QCD_3. In the $\mathcal{N}=1^*$ theory such states exist, and can potentially even be lighter than the 0^{++} glueball which corresponds to the dilaton [31]. Although the dual background presented in [1] is not a product space with an SO(6) isometry, it becomes one near the boundary. We can thus still use the spectrum derived in [32] to enumerate the SUGRA modes and classify them according to their spin and other quantum numbers. In principle, the non-trivial background fields can cause mixing between SUGRA modes. In such a case the 0^{++} glueball does not correspond simply to the dilaton, and one has to find combinations of SUGRA modes that are eigenstates of the system of linearized type-IIB equations. We will not attempt to find such a combination here. Note, however, that because the background is asymptotically $AdS_5 \times S^5$, all possible mixings will be suppressed in the UV, where the classical value of the dilaton is constant. Instead we will consider a fictitious minimally coupled scalar ψ that has the following action in the Einstein frame:

$$S_\psi \sim \int d^{10}x \sqrt{g_E}\,(\partial\psi)^2\,, \tag{5.1}$$

In a generic vacuum state where SU(N) is broken down to SU(p) we expect to find glueball states, as well as W-bosons and monopoles. Consider the background of p D5-branes. The metric and dilaton (2.8) can also be written as

$$ds^2 = Z^{-1/2}dx_{0123}^2 + Z^{1/2}\left[du^2 + u^2 d\theta^2 + u^2\sin^2\theta d\Omega_y^2\right] + \Omega^{1/2}Z^{1/2}u^2\cos^2\theta d\Omega_w^2\,, \tag{5.2}$$

$$e^{2\phi} = g^2\Omega^{1/2}\,, \quad Z = \frac{R^4}{u^4 - 2r_0^2u^2\cos 2\theta + r_0^4}\,, \quad r_0 = \frac{m\alpha'\pi N}{p}\,, \quad \Omega = \left[\frac{\rho_-^2}{\rho_-^2 + \rho_c^2}\right]^2, \tag{5.3}$$

where we introduced the coordinates $u^2 = y^2 + w^2$ and $\tan\theta = y/w$. Note that since both y and w are radii and therefore positive, θ takes values in $[0, \pi/2]$.

Consider the wave equation for ψ,

$$\partial_\mu\left[\sqrt{g_E}\, g_E^{\mu\nu}\,\partial_\nu\right]\psi = 0\,. \tag{5.4}$$

Explicitly, the equation is

$$\left[\partial_u^2 + \frac{5}{u}\partial_u + \frac{1}{u^2}(\partial_\theta^2 + 4\cot 2\theta\,\partial_\theta) - k^2 Z(u,\theta)\right]\psi = 0\,, \tag{5.5}$$

where k^μ is the 4-momentum along x_{0123}. The field theory asymptotic states associated with ψ have masses $M^2 = -k^2$.

In order to get a normalizable ψ we need to impose boundary conditions at the boundary of the (u,θ) plane [28, 29, 30]. The wave function should obey $\psi(\infty,\theta) = 0$, and there is an additional singular point at $(r_0, 0)$. We should also specify boundary conditions at $\theta = 0$ and $\theta = \pi/2$. Since both w and y are radii of three dimensional spaces, smoothness of the wave function at their origins implies $\partial_w\psi(w = 0, y) = \partial_y\psi(w, y = 0) = 0$. It is convenient to continue θ to $[0, 2\pi]$ by reflection, $y \to -y$, $w \to -w$, adding a singular point at (r_0, π) so that the wave function is automatically even with respect to these reflections, and demand $\psi(\infty, \theta) = 0$ for the enlarged range of θ. We should also impose $\psi = 0$ at the two singular points in order for the solution to be normalizable.

We can easily show that ψ has no normalizable zero mode. If one looks for a solution with $k = 0$ the equation (5.5) separates. The nontrivial eigenvalues of the operator $\partial_\theta^2 + 4\cot 2\theta\partial_\theta$ are $\alpha_n = -4(n^2 - 1)$ where $n \geq 1$ is an integer. Therefore, equation (5.5) reduces to

$$\left[\partial_u^2 + \frac{5}{u}\partial_u + \frac{\alpha_n}{u^2}\right]\psi = 0 \tag{5.6}$$

and it can be seen that there is no solution $\psi(u)$ that obeys all boundary conditions. In order to prove the existence of a mass gap one should in principle repeat such arguments for all the possible SUGRA modes, after the wave equations of type IIB are diagonalized.

The mass scale for the asymptotic states that one gets from (5.5) is

$$M^2 \sim \frac{r_0^2}{R^4} \sim \frac{m^2N^2}{gNp^2} = m^2\frac{q}{gp}\,. \tag{5.7}$$

To the extent that ψ is a good approximation of one of the fields in type-IIB SUGRA, the above mass scale is that of the glueball states corresponding to that field. The ratio gp/q is exactly the controlling parameter introduced in [1]. For weak coupling (small g) where the p D5-branes background is valid the mass scale is therefore large. At strong coupling one should use the q NS5-branes background. The wave equation is the same, but now $r_0 = m\alpha'\pi gN/q$. This leads to the following scale

$$M^2 \sim \frac{r_0^2}{R^4} \sim \frac{m^2g^2N^2}{gNq^2} = m^2\frac{gp}{q}\,. \tag{5.8}$$

Again, the ratio gp/q is large when the q NS5-branes background is valid. These results are consistent with, and generalize, the analysis carried out in [1] for a single D5 and for a single NS5-brane. As explained in [1], in the q NS5-branes background there is a throat region, and it is not clear whether any of the arguments above are valid.

6. Global symmetries and $\mathcal{N} = 0^*$ theory

In this section we consider the route of supersymmetry breaking from the $\mathcal{N} = 4$ CFT, through $\mathcal{N} = 1^*$, down to the non-supersymmetric $\mathcal{N} = 0^*$, concentrating on the global symmetries of the theories and their manifestation in the string theory duals.

JHEP03(2001)013

- $\mathcal{N} = 4$ SYM.
As mentioned above, the superpotential (2.1) manifestly exhibits a U(3) R-symmetry. Writing the scalar fields as
$$\phi_i = \frac{A_i + iA_{i+3}}{\sqrt{2}} \tag{6.1}$$
the scalar potential is
$$V \propto \sum_{i,j} \mathrm{tr}\left([A_i, A_j]^2\right), \tag{6.2}$$
with a manifest SO(6) symmetry. The symmetry is also visible in the dual description since it is a part of the isometry of the $AdS_5 \times S^5$ metric. It is important to note that in this case there are no additional background fields that are not invariant under this symmetry.

- $\mathcal{N} = 1^*$.
In the field theory picture the perturbation (2.2) breaks the R-symmetry to U(1). This symmetry which transforms the gauginos is anomalous and is further broken by instantons to $\mathbb{Z}_{2N}$. In [33] it was shown in a different context that in the SUGRA picture worldsheet instantons break the $\mathrm{U}(1)_R$ symmetry associated with a shift symmetry of one of the S^3 Euler angles. Due to a flux of a B field the worldsheet instantons produce a phase that is proportional to N times the angle so only shifts of $2\pi n/N$ are allowed, namely, a breaking of the $\mathrm{U}(1)_R$ down to $\mathbb{Z}_{2N}$. A mechanism of a similar nature may also apply in our case.

  In the equal masses case, the scalar potential
$$V \propto \sum_{i,j} \mathrm{tr}\left([\phi_i, \phi_j][\phi_i, \phi_j]^\dagger\right) + m^2 \sum_i |\phi_i|^2 + m \sum_{ijk} \epsilon^{ijk} \left([\phi_i, \phi_j]\phi_k^\dagger + [\phi_i, \phi_j]^\dagger \phi_k\right) \tag{6.3}$$
exhibits an SO(3) global symmetry. However, as discussed in section 2, this symmetry mixes with the SU(N) local gauge symmetry, and each vacuum possesses a new SO(3) symmetry.

  It is presumably this new SO(3) symmetry which is manifest as the isometry of the SUGRA dual of the $\mathcal{N} = 1^*$ theory, as we now proceed to show. Considering the metric (2.8), the isometry group seems to be SO(3) × SO(3). However, this is not the whole picture. Unlike the $AdS_5 \times S^5$ and some of its relatives, in the $N = 1^*$ there is the additional G_3 background field whose "isometries" have to be examined separately. Since it is an important point, we show now explicitly that in fact G_3 is invariant only under the diagonal subgroup $\mathrm{SO}(3)_z$ of $\mathrm{SO}(3)_w \times \mathrm{SO}(3)_y$. The field G_3 obeys
$$*_6 G_3 - iG_3 = Z'(w^2, y^2) T_3\,, \tag{6.4}$$

where T_3 is given by (2.5) with $m_1 = m_2 = m_3, m_4 = 0$. The metric and the function Z' are invariant under $\mathrm{SO}(3)_w \times \mathrm{SO}(3)_y$. The Hodge operator $*_6$ is defined relative to the *flat*, SO(6) invariant metric. In addition, the Bianchi identity is, to lowest order, $dG_3 = 0$, and in the next order of the perturbation there is a magnetic source, $dG_3 = J_4$, with $J_4 = J_4(w^2, y^2)$ also invariant under $\mathrm{SO}(3)_w \times \mathrm{SO}(3)_y$. The invariance of G_3 is therefore dictated by that of T_3. It can easily be seen that T_3 is only invariant under the diagonal subgroup $\mathrm{SO}(3)_z$ of $\mathrm{SO}(3)_w \times \mathrm{SO}(3)_y$, in which $z_i = \frac{w_i + iy_i}{\sqrt{2}}$ and $\bar{z}^i = \frac{w^i - iy^i}{\sqrt{2}}$ transform in the same manner. Explicitly, let us consider the orthogonal transformations which are the exponentiations of (2.7), keeping w_3 fixed, and taking, for any angle θ,

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \longmapsto \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \tag{6.5}$$

and similarly for y_i, therefore also for $z_i, \bar{z}_i$. We easily see that both the two-form wedged with dz_3 and the one wedged with $d\bar{z}_3$ are invariant, and therefore so is T_3. It can also be easily seen that there are no transformations outside $\mathrm{SO}(3)_z$ keeping T_3 invariant. Therefore, this should also be the invariance group of G_3, and the global symmetry of the SUGRA solution. Alternatively, The explicit solution of G_3 given in [1] can be checked out to show the same invariance.

- $\mathcal{N} = 0^*$, mass to the gaugino alone.
  Supersymmetry may be completely broken by giving mass to the gaugino. This can be done by perturbing the superpotential [34]

$$\Delta W = M_4 S\,, \tag{6.6}$$

where S is the glueball chiral superfield (including $\mathrm{tr}(\lambda_4\lambda_4)$) and M_4 is a chiral superfield whose F-term is m_4. Obviously, such a perturbation breaks explicitly the $\mathrm{U}(1)_R$ symmetry. The m_4 component is the part of the $\overline{\mathbf{10}}$ representation which is a singlet under the SU(3) but carries a charge under the $\mathrm{U}(1)_R$. If we assign a mass to the gaugino, keeping all the other fields massless at tree level, the classical global symmetry is SU(3). This chiral symmetry is claimed to be broken by the condensate $\langle\lambda_i\lambda_i\rangle$ of the chiral superfield fermions [35, 37].

The corresponding SUGRA perturbation is turned on by the last term in (2.5). A mass term only for the gaugino means that the terms proportional to m are switched off. This setup was shown in [36] to be outside the range of validity of the Polchinski-Strassler solution. Once supersymmetry is broken, the scalars are no longer protected and will inevitably acquire masses through radiative corrections.

- $\mathcal{N} = 0^*$, mass to the scalars alone.
  An alternative method to break supersymmetry while maintaining the classical $\mathrm{U}(1)_R$ symmetry is giving masses to the scalars. The traceless part gives mass to $\mathrm{tr}(\phi_i\phi_j - \frac{1}{6}\phi^2\delta_{ij})$, a chiral primary operator transforming in the **20'** of SO(6), which corresponds [32, 38] to the SUGRA field S_2 which is a linear combination of the four-form and of the trace of the metric on S^5. In the notation of [1] it is μ_{mn}. We note that such a mass term gives rise to tachyonic modes. This may indicate that the theory does not possess a vacuum state. One can contemplate that this problem may be cured non-perturbatively, by generating a positive potential leading to a stable minimum, corresponding to a true ground state. However, instantons do not seem to create such a potential. Moreover, for large fields we expect the semi-classical approximation to hold. Another way this problem can be solved is by adding the trace. The trace part $\mathrm{tr}(\phi_i\phi_i)$ is not a chiral operator and corresponds to a stringy mode. Clearly we do not know how to implement this alternative or how to check it. Note that this perturbation alone does not involve the T_3 and thus the creation of 5-branes via the Myers effect is absent here. At the tree level there is an SU(4) symmetry that protects the masslessness of all the fermions.

- $\mathcal{N} = 0^*$, mass to the chiral superfields and to the gaugino.
  Obviously, any perturbation caused by turning on either m_4 or μ_{mn}, combined with m, breaks supersymmetry completely. The global symmetries associated with these models follow from those of the previous $N = 0^*$ models. For instance, the symmetry of the theory with m and m_4 turned on is SO(3). These theories were considered in [36] and the string dual, practically identical to the one in [1], indeed possesses the SO(3) symmetry.

7. Summary and discussion

Supergravity duals of large-N gauge theories have provided a useful laboratory to "measure" gauge invariant properties in the large λ regime. In particular, Wilson loops were computed in about a dozen of SUGRA setups [39] and were shown to admit the behavior anticipated from gauge dynamics. Apparently, that is not the case for the dual of the $\mathcal{N} = 1^*$ model. A naive calculation of the minimal area of a string worldsheet in the metric of the model yields "wrong" expectation values for the Wilson loops (for instance, for the vacuum associated with the p D5-branes). The reason for the failure of the naive approach in the present case is obviously twofold. First, the curvature of the metric diverges and the SUGRA approximation ceases to hold. Moreover, there is an additional non-trivial background field, the magnetic three-form. In fact the coupling of the string also to a $B^{(NS)}_{\mu\nu}$ occurs in SUGRA duals of certain field theories on non-commutative spacetime [40, 41, 42]. In those cases the coupling is well understood and the extraction of the Wilson loops indeed produced the anticipated picture. Unfortunately, at present the determination of the full string action in the $\mathcal{N} = 1^*$ dual is beyond our abilities. However, as was shown in section 3, the incorporation of the world volume gauge dynamics on the unwrapped part of the five-branes leads to a picture that at least qualitatively is in accordance with the properties of the field theory vacua.

A natural question that arises is what should the string action look like had we known how to incorporate the coupling to the full background. Again let us address the case where the SU(N) symmetry is broken down to SU(p). The string tension for such a case may have a factor of the form $\sin(2\pi k/p)$ where k is the p-ality of the external quarks. Such a behavior was detected in two-dimensional systems [43] as well as in MQCD [44]. An interesting open question is whether a similar structure occurs also in the $\mathcal{N} = 1^*$ dual.

A challenging puzzle (that has been already mentioned in [1]) is the fact that there are multiple valid descriptions for the same vacuum. Contrary to [1], it seems that in the SUGRA approximation, taking into account also brane dynamics, there is more than one description corresponding to each vacuum and regime of g. We have explained why this result is perplexing, and have discussed several possible resolutions of the problem. This question certainly deserves further study.

The $\mathcal{N} = 1^*$ theory serves as a bridge between theories which are rich with supersymmetries and typically are non confining, and those with $\mathcal{N} = 1$ or less that have confining phases. Recently, important steps toward the construction of a dual string model of pure $\mathcal{N} = 1$ SYM theory were made [45, 33]. It would be interesting to compute Wilson-'t Hooft loops, analyze the vacuum structure, determine the glueballs spectrum etc. in these novel SUGRA backgrounds.

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A. Counting of brane configurations

First we prove that the number of pairs $(c'', d'') \in \mathbb{Z}_N \times \mathbb{Z}_N$ mutually prime modulu N is $\phi(N)\sigma_1(N)$, where $\phi(N)$ is Euler's totient function, counting the number of $1 \leq i \leq N$ relatively prime to N, and $\sigma_1(N)$ is the sum of the divisors of N. The reason is as follows. Denote $v = \gcd(c'', N)$, so that v can be any divisor of N, and write $c'' = vw$. w is mutually prime to N/v, so the number of its possible values, which is also the number of possible c'', is $\phi(N/v)$. Now, d'' must be mutually prime with v, so the number of its possible values is $\frac{N}{v}\phi(v)$. N is square-free, so v and N/v are mutually prime, therefore by a fundamental property of Euler's function, the number of pairs (c'', d'') for a given v is $\phi(N/v) \cdot \frac{N}{v}\phi(v) = \frac{N}{v}\phi(N)$. Finally, the number of (c'', d'') pairs for any v is $\sum_{v|N} \frac{N}{v}\phi(N) = \phi(N)\sum_{v'|N} v' = \phi(N)\sigma_1(N)$.

Further we should select p' and appropriate (r'', s'') for a given (c'', d'') pair. As there are in this case exactly p' physically distinct (r'', s'') pairs, this gives another factor of $\sum_{p'|N} p' = \sigma_1(N)$, so the total number of vacua representations is $\phi(N)\sigma_1(N)^2$. The number of distinct quantum massive vacua is $\sigma_1(N)$, as was explained in section 2, and every vacuum is represented the same number of times, as argued in section 4. Therefore, the number of representations for each vacuum is $\phi(N)\sigma_1(N)$, exactly as the number of (c'', d'') pairs.

Indeed, we find that each vacuum is obtained exactly once from every (c'', d'') pair, with an appropriate $p', (r'', s'')$. This is easily seen for the confining vacuum, having screened charges subgroup $P = \langle(0, 1)\rangle$. For this vacuum, $q' \equiv \gcd(c'', N)$ and $\gcd(r'', N) = p'$, so there are no screened particles with non zero electric charge modulu N. As also explained in section 4, this result can, by the action of $\mathrm{SL}(2, \mathbb{Z}_N)$, be applied also to all the other massive vacua for the square-free N case.

The distribution of representations regarding p' is also egalitarian, each vacuum obtained $p'\phi(N)$ times from each p'.

Elementary number theory shows that for square-free N, $\phi(N) = \prod_{p|N}(p - 1)$ and $\sigma_1(N) = \prod_{p|N}(p + 1)$. We shall now give an example for the smallest non-trivial case, $N = 6$, for which the number of representations for each vacuum is already $\phi(6)\sigma_1(6) = 2 \cdot 12 = 24$. We choose to display results for the non-oblique SU(3) vacuum, for which the subgroup of screened charges is $P = \{(0, 0), (0, 2), (0, 4), (3, 0), (3, 2), (3, 4)\}$. We repeat that the choice of (r'', s'') for a given (c'', d') is not unique, as (r'', s'') and $(r'' + p'c'', s'' + p'd'')$ $(\mathrm{mod} N)$ yield the same generator (after multiplication by q'). However, the physical outcome is identical for two (or more) such (r'', s'') pairs.

The first row of table 2 corresponds to $(c, d) = p' \cdot (c'', d'') = 3 \cdot (0, 1) = (0, 3)$, or to the three D5-brane configuration of [1]. The third row corresponds to $(c, d) = p' \cdot (c'', d'') = 2 \cdot (1, 0) = (2, 0)$, or to the two NS5-brane configuration of [1], while the tenth row is the $(2, 3)$ 5-brane solution introduced in section 4.

Now we move to make exact the claim of section 4 that two valid 5-brane descriptions of the same vacuum are small multiples of the same basic configuration. We assume that the two descriptions have the same p'. In fact, we shall take $p' = 1$ in the following, as the general case is very similar. We pick some p (with $p \cdot q = N$) and demand that both descriptions involve less than q NS5-branes and less than p D5-branes. This is natural for valid descriptions, for we know [1] that p D5-branes and q NS5-branes cannot be both valid descriptions for the same value of g. Let

	(c'', d'')	(r'', s'')	p'		(c'', d'')	(r'', s'')	p'
1	$(0,1)$	$(5,0)$	3	13	$(3,2)$	$(4,3)$	6
2	$(0,5)$	$(1,0)$	3	14	$(3,4)$	$(2,3)$	6
3	$(1,0)$	$(0,1)$	2	15	$(3,5)$	$(4,3)$	6
4	$(1,1)$	$(2,3)$	6	16	$(4,1)$	$(5,0)$	3
5	$(1,2)$	$(4,3)$	6	17	$(4,3)$	$(1,1)$	1
6	$(1,3)$	$(0,1)$	2	18	$(4,5)$	$(1,3)$	3
7	$(1,4)$	$(2,3)$	6	19	$(5,0)$	$(0,5)$	2
8	$(1,5)$	$(4,3)$	6	20	$(5,1)$	$(2,3)$	6
9	$(2,1)$	$(5,0)$	3	21	$(5,2)$	$(4,3)$	6
10	$(2,3)$	$(5,5)$	1	22	$(5,3)$	$(4,5)$	2
11	$(2,5)$	$(1,3)$	3	23	$(5,4)$	$(2,3)$	6
12	$(3,1)$	$(2,3)$	6	24	$(5,5)$	$(4,3)$	6

Table 2: List of all configurations giving rise to the non oblique SU(3) vacuum of SU(6) $\mathcal{N} = 1^*$ theory.

the two 5-branes be (c_1, d_1) and (c_2, d_2). The counting argument given above shows that $(c_2, d_2) \equiv n \cdot (c_1, d_1) \pmod N$ for some n mutually prime with N, but a priori the magnitude of n is unknown to us. Multiplying the first element of this equation by d_1, the second by d_2, and subtracting, we get $c_2 d_1 - c_1 d_2 \equiv 0 \pmod N$. But as by assumption, $0 \leq c_1,\ c_2 < q$ and $0 \leq d_1,\ d_2 < p$, we must have $c_2 d_1 - c_1 d_2 = 0$. Therefore $c_2/c_1 = d_2/d_1$, and both solutions are indeed (small) multiplications of some basic one.

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JHEP03(2001)013